Problem Set: Fiscal Policy, Government Debt, and Capital Crowding Out

Advanced Macroeconomics — Dr Lei Pan — Total: 100 Marks

Instructions. Answer all questions. Show all mathematical derivations clearly. Answers that only state final results receive limited credit. Unless otherwise stated, all variables are positive and all rates are expressed in decimal form.

Question 1: Fiscal Policy, Debt Dynamics, and Intertemporal Solvency [Total: 50 marks]

During recessions and wars, governments often run large deficits. During expansions, they may be expected to run surpluses to stabilise public debt. This question studies debt accumulation, debt-to-GDP dynamics, and the intertemporal government budget constraint.

Let D_t denote the nominal stock of public debt at the end of period t, i_t the nominal interest rate, G_t government purchases, Tr_t transfers, T_t tax revenue, P_tY_t nominal GDP, g_t real GDP growth, and π_t inflation. Suppose money finance is unavailable.

(a) Starting from the flow government budget constraint, derive

$$D_t = (1 + i_t)D_{t-1} + G_t + Tr_t - T_t.$$

Define the primary balance as

$$PB_t \equiv T_t - G_t - Tr_t.$$

Then derive the exact debt-to-GDP law of motion:

$$d_t = \frac{1 + i_t}{(1 + g_t)(1 + \pi_t)} d_{t-1} - pb_t,$$

where

$$d_t \equiv \frac{D_t}{P_tY_t}, \qquad pb_t \equiv \frac{PB_t}{P_tY_t}.$$

Finally derive the approximate law

$$\Delta d_t \approx (r_t - g_t)d_{t-1} - pb_t,$$

where r_t is the real interest rate. **[12 marks]**

(b) Suppose r and g are constant and define

$$R \equiv \frac{1 + r}{1 + g}.$$

Assume the primary balance ratio is constant at $pb_t = p$. Solve the first-order difference equation

$$d_t = Rd_{t-1} - p.$$

Derive the primary balance required to stabilise the debt ratio at a target $\bar{d}$. Explain the cases $r > g$, $r = g$, and $r < g$. **[10 marks]**

(c) Consider a country with initial debt ratio

$$d_0 = 0.90,$$

nominal interest rate

$$i = 0.05,$$

inflation

$$\pi = 0.02,$$

real GDP growth

$$g = 0.015,$$

and a primary deficit equal to 2% of GDP, so that

$$pb = -0.02.$$

Using the exact debt-to-GDP equation, compute d_1, d_2, d_3, d_4, d_5. Interpret the fiscal sustainability implication. **[10 marks]**

(d) Consider a two-period economy with no money finance and no transfers. The government begins period 1 with debt D_1. Derive the two-period intertemporal government budget constraint:

$$T_1 - G_1 + \frac{T_2 - G_2}{1 + i} = (1 + i)D_1.$$

Suppose

$$i = 0.04, \qquad D_1 = 0.30Y, \qquad G_1 = 0.22Y, \qquad G_2 = 0.20Y, \qquad T_1 = 0.18Y.$$

Compute the required value of T_2/Y if the government must end period 2 with zero debt. **[10 marks]**

(e) Starting from the national income identity for an open economy,

$$Y = C + I + G + EX - IM,$$

derive

$$I = (Y - T - C) + (T - G) + (IM - EX).$$

Use this identity to explain analytically why a larger government deficit may crowd out investment, and state two mechanisms that may offset full crowding out. **[8 marks]**

Question 2: Government Debt, Saving, and Ricardian Equivalence in an OLG Economy [Total: 50 marks]

Consider a two-period-lived overlapping-generations economy. Population is constant. Each individual receives y_1 goods when young and y_2 goods when old. There is no money. Individuals save by holding physical capital or one-period government bonds. Both assets earn the same gross return $1 + r$. The young in period t pay a lump-sum tax $\tau_{1,t}$, and the old in period $t + 1$ pay $\tau_{2,t+1}$.

The individual budget constraints are

$$c_{1,t} + s_t = y_1 - \tau_{1,t},$$
$$c_{2,t+1} = y_2 + (1 + r)s_t - \tau_{2,t+1}.$$

Government bonds per young person are b_t, physical capital per young person is k_t, and total private saving satisfies

$$s_t = k_t + b_t.$$

(a) Suppose preferences are

$$U(c_{1,t}, c_{2,t+1}) = \ln c_{1,t} + \beta \ln c_{2,t+1}, \qquad \beta > 0.$$

Derive the lifetime budget constraint, the Euler equation, optimal $c_{1,t}$ and $c_{2,t+1}$, and the saving function

$$s_t = s(y_1, y_2, \tau_{1,t}, \tau_{2,t+1}, r, \beta).$$

Then derive the signs of

$$\frac{\partial s_t}{\partial \tau_{1,t}} \quad \text{and} \quad \frac{\partial s_t}{\partial \tau_{2,t+1}}.$$

[14 marks]

(b) The government cuts taxes on the young in period t by $x > 0$:

$$\tau_{1,t}^N = \tau_{1,t} - x,$$

with no change in current government expenditure. In Case 1, the extra debt is rolled over and is eventually repaid by future generations, not by generation t. Derive

$$\Delta b_t = x, \qquad \Delta b_{t+j} = x(1 + r)^j \quad \text{for } j \geq 1$$

before final retirement. Under log utility, compute

$$\Delta c_{1,t}, \qquad \Delta c_{2,t+1}, \qquad \Delta s_t, \qquad \Delta k_t.$$

Show that government debt crowds out capital. **[12 marks]**

(c) In Case 2, the same tax cut x for the young in period t is repaid by taxing the same generation when old in period $t+1$. Derive the required tax increase

$$\Delta \tau_{2,t+1} = (1 + r)x.$$

Then prove that lifetime wealth, consumption, and capital are unchanged:

$$\Delta c_{1,t} = 0, \qquad \Delta c_{2,t+1} = 0, \qquad \Delta k_t = 0.$$

Explain why private saving rises one-for-one with government debt. **[10 marks]**

(d) Now drop log utility. Suppose only that both $c_{1,t}$ and $c_{2,t+1}$ are normal goods. Prove that in Case 1,

$$0 < \Delta s_t < x$$

and therefore

$$\Delta k_t < 0.$$

Then explain why in Case 2 Ricardian equivalence holds even without log utility. **[8 marks]**

(e) Suppose households internalise only a fraction $\mu \in [0, 1]$ of the future tax burden generated by the tax cut. Under log utility, the effective present-value wealth gain is

$$(1 - \mu)x.$$

Derive

$$\Delta s_t = \frac{\beta + \mu}{1 + \beta} x$$

and

$$\Delta k_t = -\frac{1 - \mu}{1 + \beta} x.$$

Interpret the two polar cases $\mu = 0$ and $\mu = 1$. **[6 marks]**